Intersection Proba. for RWs.

(1) long Range:

<u>prop.</u> For $p \in \mathcal{P}_d$. There exists. c_1, c_2. for $\forall n \geq 2$:

$$c_1 \phi(n) \leq \mathbb{P}(S[0,n] \cap S[2n,3n] \neq \emptyset)$$

$$\leq \mathbb{P}(S[0,n] \cap S[2n,\infty) \neq \emptyset) \leq c_2 \phi(n)$$

$$\phi(n) = \begin{cases} 1. & d < 4 \\ (\log n)^{-1}. & d = 4 \\ n^{(4-d)/2}. & d > 4 \end{cases}$$

<u>Cor.</u> For BM in $\mathbb{R}^d$. $\mathbb{P}(B[0,1] \cap B[2,3] \neq \emptyset)$

$$\begin{cases} > 0. & d \leq 3. \\ = 0. & d \geq 4. \end{cases}$$

<u>Pf:</u> Donsker's law. set $n \to \infty$.

<u>Pf:</u> WLOG. $d \geq 3$. the walk is aperiodic.

Set $J_n = \sum_{j=0}^{n} \sum_{k=2n}^{3n} I\{S_j = S_k\}$.

$K_n = \sum_{j=0}^{n} \sum_{k \geq 2n} I\{S_j = S_k\}$.

estimate $\mathbb{E}(J_n)$. $\mathbb{E}(J_n^2)$. by 2^{nd} moment.

analysis: $\mathbb{P}(J_n > 0) \geq \mathbb{E}(J_n)^2 / 4\,\mathbb{E}(J_n^2)$.

to generate the lower bound.

For upper bdd: $\mathbb{P}(K_n \geq 1) \leq \mathbb{E}(K_n)$.

(2) Short Range:

$S, S^1, S^2 \cdots$ are i.i.d RW starts at 0 with pt D_d distribution. $T_\lambda^i \overset{i.i.d}{\sim} \text{Geo}(1-\lambda)$, $\forall i$.

Denote $\lambda_n = 1 - n^{-1}$.

Next, we estimate: $\mathbb{P}(S[0,n] \cap S^1(0,n] = \emptyset)$.

Lemma. $Q(\lambda) =: \sum_{q} \mathbb{P}^{(0,q)}(S[0,T_\lambda] \cap \tilde{S}[0,T_\lambda] \neq \emptyset) \overset{n \to \infty}{\asymp} \begin{cases} n^{\frac{d}{2}}, & d < 4 \\ n^2/\log n, & d = 4 \end{cases}$

Prop. $V_\lambda = \{0 \notin S^1(0, T_\lambda^1]\}$. Then: we have,

$\mathbb{P}(V_\lambda \cap \{S(0,T_\lambda] \cap (S^1(0,T_\lambda^1] \cup S^2(0,T_\lambda^2]) = \emptyset\}) = (1-\lambda)^2 Q(\lambda)$.

Cor. For $d = 2, 3, 4$.

$$\mathbb{P}(S(0,n] \cap (S^1(0,n] \cup S^2(0,n]) = \emptyset)$$

$$\asymp \mathbb{P}(S(0,T_{\lambda_n}] \cap (S^1(0,T_{\lambda_n}] \cup S^2(0,T_{\lambda_n}]) = \emptyset)$$

$$\asymp (1-\lambda_n)^2 Q(\lambda_n) \asymp \begin{cases} n^{(d-4)/2}, & d = 2, 3 \\ (\log n)^{-1}, & d = 4. \end{cases}$$

Rmk: When $d > 4$, the prob. may not converge to 0 as $n \to \infty$.